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Quantum black hole inflation

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Abstract

In this paper we follow a new approach for particle creation by a localized strong gravitational field. The approach is based on a definition of the physical vacuum drawn from Heisenberg uncertainty principle. Using the fact that the gravitational field red-shifts the frequency modes of the vacuum, a condition on the minimum strength of the gravitational field required to achieve real particle creation is derived. Application of this requirement on a Schwartzchild black hole resulted in deducing an upper limit on the region, outside the event horizon, where real particles can be created. Using this regional upper limit, and considering particle creation by black holes as a consequence of the Casimir effect, with the assumption that the created quanta are to be added to the initial energy, we deduce a natural power law for the development of the event horizon, and consequently a logarithmic law for the area spectrum of an inflating black hole. Application of the results on a cosmological model shows that if we start with a Planck-dimensional black hole, then through the process of particle creation we end up with a universe having the presently estimated critical density. Such a universe will be in a state of eternal inflation.

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I. Introduction

Since the pioneering works of Parker [1] and of Zeldovich and Starobinsky [2], large number of papers has been published on the the subject of particle creation in curved spacetime. In this respect the celebrated paper of Hawking [3] marked a milestone in an approach relating gravity with quantum field theory and thermodynamics. It was discovered that black holes can creat particles and therefore radiate energy which was found to have a black-body spectrum, this was later called the ‘Hawking effect’. Davies [4] showed that a uniformly accelerated mirror in Rindler space creates thermal spectrum. Also, it was shown by Unruh [5] that even a uniformly accelerated observer in Minkowski vacuum experience a thermal bath of created particles, this was later called the ‘Unruh effect’.

The detailed analysis of particle creation by a localized gravitational field (or by accelerated mirrors or Minkowski observers) stems, in essence, from the fact that the gravitational field (or accelerated systems) red-shifts the outgoing modes that are leaving the collapsing body in the Hawking effect. In the case of moving mirror, the Doppler shift reproduces the same effect that the gravitational field produces on the field modes. This is the standard explanation given to the phenomena of particle creation by gravitational field, (for more details see ref. [6] and for more recent review see ref. [7]). However, the direction of the out-going modes is another matter, which is normally related with the boundary conditions considered [7]. Also, whether the spectrum of the emitted radiations will be thermal or not is a matter of system conditions and the special ansatz employed [8].

Before the Hawking discovery, Price [9] studied the nonspherical perturbations of relativistic collapse in presence of a scalar field. He showed that at certain values of r , outside the event horizon there exist potential barriers with respect to the scalar waves. The positions of these barriers depend on the value of the multipole l . for $l = 0$ the position of the potential barrier is at $r = \frac{8}{3}M$, for $l = 1$ it is at $r = 2.88M$, for $l = 3$ it is at $r = 2.95M$, and as $l \longrightarrow \infty$ the position of the potential barrier $r \longrightarrow 3M$.

Nugayev and Bashkov [10] considered particle creation by black holes as a consequence of the Casimir effect. They showed that the gravitational field of the black hole produces a potential barrier outside the event horizon at $r = 3M$. In a later paper Nugayev [11] showed that the black hole radiation

is due to the interaction of virtual particles with the "cavity" formed between the event horizon and the potential barrier at $r = 3M$. This result established a close relationship between the Casimir effect and particle creation by black holes.

This relationship was explicitly treated by Anderson et al [12] who studied the semiclassical corrections of the scalar field to the Schwartzchild metric. Using a general form for the energy-momentum tensor which was calculated by the authors in an earlier work [13], the authors found some interesting properties for the vacuum expectation value of the components of the energy-momentum tensor in the vicinity of the event horizon of a Schwartzchild black hole. For example they found that the energy density of the scalar field at the horizon will be negative for all values of the curvature coupling with values $\xi < 4/15$. This means that for $\xi = \frac{1}{6}$ (the conformally coupled field), and for $\xi = 0$ (the minimally coupled field), the energy density is negative. On the other hand, they find that the energy density become positive for large values of ξ . The energy density is found to be everywhere positive outside the horizon for values of ξ in the range $\frac{4}{15} < \xi < 1.2515$. However, for large value of ξ outside the horizon, Anderson et al. found a point at which the energy density is independent of ξ .

Berezin, Boyarsky and Neronov [14] considered a massive self-gravitating shell as a model for the collapsing body and a null self-gravitating shell as a model for the Hawking radiation. They showed that the mass-energy spectra for the body and the radiation do not match. They resolved this situation by considering the resultant effects of the outgoing and incoming radiation modes together. This consideration led them to the important result that the quanta of radiation created by a black hole come in pairs, one emitted to infinity and the other falls on the black hole. This will obviously cause a change of the inner structure of the black hole resulting, according to their conclusion, in a Bekenstein-Mukhanov [15] spectrum for large masses. This conclusion will be discussed further in this paper.

Recently Schützhold [16] suggested a canonical particle definition via the diagonalization of the Hamiltonian for quantum fields in specific curved spacetimes. Within his suggested approach radial in-going or out-going Minkowski particles do not exist. An application of this approach to the Rindler metric recovers the Unruh effect. The application of the same approach on particle creation by a black hole shows a genuine dependance of

the Hawking temperature on the dynamics of the collapse. Furthermore it was shown through the evolution of the vacuum expectation value of the energy-momentum tensor that there is no late-time Hawking radiation and therefore no black hole evaporation.

These investigations, and much of the controvercies over the subject brings up the need for more elaborate and simple approach to deduce particle creation by gravitational field. Physically the idea is feasible in the light of basic knowledge gained from quantum mechanics and general relativity. Since time is dilated by the presence of gravitational field (or equivalently the energy spectrum is red-shifted) with respect to a Minkowskian frame of reference the law of conservation of energy (or the principle of general covariance in more accurate terms) should allow for particle creation in pairs that should uphold the conservation laws.

Throughout all the original derivations of particle creation by black holes or moving boundaries we notice that the main role is played implicitly by the action of the red-shift of the outgoing frequency modes with respect to the in-comming frequency modes. This is just another facet of time-dilation experienced by events taking place in non-inertial frames of reference. This effect is best clarified by the response function where the red-shift is made promenant [6].

In this paper we consider a new approach to deduce particle creation by a localized strong gravitational field. The approach is based on a definition of physical vacuum, (cosequently a definition of real particle) based on Heisenberg uncertainty principle. The gravitational red-shift of the frequency modes is used to determine the minimum strength of the gravitational field needed to achieve the minimum amount of red-shift required to convert virtual particles into real ones. Accordingly, a regional upper limit in the vicinity of the Schwartzchild black hole is deduced, below which only, real particle creation can take place. This upper limit is further utilized to define a Casimir second surface in a system where the first surface is assumed to be the event horizon itself. The system will induce a non-zero Casimir energy which is assumed to be added to the initial total energy of the black hole. This addition will cause a change in the internal structure of the black hole, consequently the area of the event horizon will increase. This mechanism will cause the black hole to expand logarithmically, i.e, to inflate indefinitely without end. An application of these results on a model black hole that is

assumed initially to have Planck dimensions leads to a black hole having a mass that is equal to the critical mass in the present universe. Accordingly we speculate that if our universe was born as a black hole then it is most possible that it has the critical mass density by now.

The rest of this paper is organized as follows: in Sec. II we present an overview of the customary definition of vacuum according to quantum field theory, and suggest an alternative definition, which may be less ambiguous, based on the general relativity and the Heisenberg uncertainty principle. In Sec.III we investigate the relation between the energy and time in presence of a localized gravitational field, through the study of the gravitational red-shift, where we deduce a lower limit on the value of the red-shift necessary for the conversion of virtual particles of the Minkowski vacuum into real particles. In Sec. IV we apply this lower limit to a Schwartzchild black hole where we deduce an upper limit on the region in the vicinity of the black hole where real particles can be created. This upper limit is found to coincide with the position of the potential barrier of Price [9]. In Sec.V we setup a Casimir system within which the creation of vacuum energy takes place, and in sec.VI a successively developing Cauchy surfaces structure is constructed to explain the development of the black hole if the created Casimir energy is to be added to the initial energy. A cosmological application is presented in sec.VII, where we find that the proposed natural mechanism which sets the black hole into an inflationary state can develop a Planck-sized black hole into a universe-sized one with the present critical density. Finally we present in sec. VIII a thorough discussion of the results in view of available related investigations, outlining at the same time further implications of our results. Throughout this paper we use the natural system of units defined by $\hbar = c = G = 1$ unless otherwise stated.

II. Definition of the Vacuum

In quantum field theory the vacuum is understood to be the state of no particle. Although the definition of vacuum in Minkowski spacetime seems to be trivial, the case is not so in curved spacetime [6,17]. In Minkowski flat spacetime the vacuum state is defined by

$$\hat{a}|0\rangle = 0, \tag{1}$$

where \hat{a} is called the annihilation operator.

The definition in (1) above leads to problems when calculating the vacuum expectation value of the Hamiltonian \hat{H} for a bound system (e.g harmonic oscillator) where it is known that

$$\langle 0|\hat{H}|0\rangle = \sum_k \frac{1}{2}\omega_k \quad (2)$$

The sum on the RHS diverges. These divergences are considered a chronic disorder in the structure of quantum field theory [18]. Usually special prescriptions are used to remove these divergences but all lack the conceptual foundations.

An alternative definition of the vacuum may be obtained by incorporating the concepts of the general theory of relativity and quantum mechanics. It is known that the Einstein field equation for empty spacetime,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 0, \quad (3)$$

admits solutions other than the Minkowski flat spacetime solution. It is notable that all non-flat solutions do contribute a non-zero vacuum energy density, (called Casimir energy) whereas the flat Minkowski spacetime contributes nothing. This suggests that a true vacuum state cannot exist in a curved spacetime, a point which was rigorously supported by the findings of Fulling [17]. Therefore, and in order to investigate the behavior of matter fields in curved spacetime, the generally adopted trend resorts to assume the existence of two asymptotically flat regions, one in the remote past (past infinity) and the other at late times (future infinity). This is the approach followed by the references cited above. However, from a general relativistic point of view, the true vacuum can only exist in the form of the Minkowski spacetime. This accurately means that a true Minkowski spacetime has to be physically empty. Therefore, in the conceptual sense, no particle should be assumed to exist in a Minkowski spacetime, and once a particle is assumed to exist, the spacetime is no longer Minkowskian.

This brings us to the situation where the vacuum needs to be re-defined. For this goal, we define the vacuum as being the state of no measurable physical energy. This means that the assumption that a state $|0\rangle$ can exist in the remote past or in the far future can be replaced by the requirement that

$$Et < 1 \tag{4}$$

where E is the total energy of the state and t is the duration through which it render itself for physical measurement.

The condition in (4) is nessessary to assure that no real particle exist, on the other hand this condition allows for virtual states to be assumed to exist. Such states are well-known in flat space quantum field theory and are used to explain some properties of forces and interactions. Therefore instead of talking about the Minkowski vacuum, we may talk about a "Heisenberg vacuum" defined by Eq. (4) where we have now a virtual particle of energy E allowed to exist within a time interval t such that Eq. (4) is satisfied.

III. Energy and Time

According to the theory of general relativity, if a frequency mode (a quanta) of frequency ω_2 is emitted in a static gravitational field at a point x_2 then this mode will be received at a point x_1 with a frequency ω_1 . The relation between the two frequencies is given by

$$\frac{\omega_1}{\omega_2} = \left[\frac{g_{00}(x_1)}{g_{00}(x_2)} \right]^{1/2}. \tag{5}$$

This correspond to a blue-shift or a red-shift of the respective frequencies depending on the relative positions of the points x_1 and x_2 . If x_1 is at the weak field point then an observer at x_2 will see the frequency coming from x_1 blue-shifted, but if x_1 is at the stronger field then the observer at x_2 will see the frequency red-shifted. Let us assume that x_1 is a point at the asymptotically flat region of the space, then we can define the red-shift parameter as

$$z = \frac{\Delta t}{t_1} = \left(\frac{g_{00}(x_1)}{g_{00}(x_2)} \right)^{1/2} - 1, \tag{6}$$

where

$$\Delta t = t_2 - t_1. \tag{7}$$

The energy of a real state at the two points will be related by Eq. (5) above.

Now let us look at the two points x_1 and x_2 not as two separate points in a given static gravitational fields but as the same point belonging to two different situations of the field. The first, x_1 , is when the field is off and the other, x_2 , is when the field is suddenly switched on. This assumption was already used by Schützhold [16] and others in the study of particle creation by black holes.

By assuming x_1 to be a point at the asymptotically flat region (i.e Minkowskian region), we are entitled to consider the states in this region as representing the true physical vacuum. Therefore, in this region we can assume that

$$E_1 t_1 < 1, \quad (8)$$

where E_1 and t_1 are the energy and the lifetime of the state respectively.

Now switching-on the gravitational field will change the status of x_1 into x_2 where real particles may exist with energy E_2 and a lifetime t_2 satisfying the relation

$$\Delta E \Delta t > 1, \quad (9)$$

where

$$\Delta E = E_1 - E_2, \quad (10)$$

and Δt is as in Eq. (7).

Now, with simple algebraic manipulations one can show that

$$\frac{\Delta t}{t_1} - \frac{\Delta E}{E_1} \gtrsim \frac{1}{E_1 t_1} + \frac{E_2 t_2}{E_1 t_1} - 1, \quad (11)$$

and by Eq. (8) we have $\frac{1}{E_1 t_1} > 1$, also by Eq. (5) $E_1 t_1 = E_2 t_2$, therefore

$$\frac{\Delta t}{t_1} > 1 + \frac{\Delta E}{E_1}. \quad (12)$$

Using the definition of the red-shift parameter z as in Eq. (6) we deduce that for real particle creation to takeplace in a given gravitational field there should be a lower limit for the value of the gravitational red-shift parameter given by

$$z > 1, \quad (13)$$

and for values of $z < 1$, only virtual particles may exist.

The condition in Eq. (13) can be expressed in terms of the gravitational potential between any two points in a gravitational field. From Eq. (6) and Eq. (13) we get

$$\left[\frac{g_{00}(x_1)}{g_{00}(x_2)} \right]^{1/2} - 1 > 1 \quad (14)$$

Thus the condition for real particle creation becomes

$$g_{00}(x_1) > 4g_{00}(x_2) \quad (15)$$

IV. The Schwartzchild black hole

Now we apply the result of the previous section to a non-rotating Schwartzchild black hole.

For a spherical star of mass M and radius R_b we have

$$g_{00}(x_2) = 1 - \frac{2M}{R_b} \quad (16)$$

If we consider the position of the observer to be at a point x_1 in an asymptotically flat region where $g_{00}(x_1) \approx 1$, then the inequality in Eq. (15) gives

$$4 - \frac{8M}{R_b} < 1, \quad (17)$$

which means that

$$R_b < \frac{8}{3}M. \quad (18)$$

In terms of the Schwartzchild radius of the star $R_s = 2M$ this condition can be written as

$$R_b < \frac{4}{3}R_s. \quad (19)$$

This condition defines a regional limit outside the event horizon, below which only, real particle creation can take place. This limit coincides with the position of the potential barrier for the S-wave part of the massless scalar field in the gravitational field of a non-rotating black hole calculated by Price [9].

The above regional limit means that the region in the vicinity of a black hole in which real particle creation can take place is confined by two Cauchy surfaces; the first is the horizon of the black hole and the second is a surface located at $R = \frac{4}{3}R_s$. In between these two surfaces, real particles can be created.

V. Particle Creation versus Casimir effect

In flat space it was found that the vacuum fluctuations of the electromagnetic field give rise to an attractive force between two parallel conducting flat plates, this creates a negative energy density inversely proportional to the fourth power of the distance between the two plates. This was discovered by Casimir [19] and was called the Casimir effect. Application of this effect in closed spacetimes (e.g the Einstein universe) has shown that it lead to a positive energy density (for example see ref. [20, 21]). Further consideration of the problem at finite temperatures resulted in calculating the finite temperature corrections which was shown to be an important driver for the thermal development of the universe when considered as a source for the Einstein field equations [22, 23].

Following Nugaev [11], we may exchange the nonrotating black hole with two spherical conductors one just outside the event horizon but very near to it, and the other to be consider just below the upper limit $\frac{4}{3}R_s$. These two surfaces will constitute concentric shells, analogous to the parallel conducting plates. Any amount of energy created through the Casimir mechanism between the two shells is assumed to be added to the total energy of the black hole, hence extending the event horizon by an amount that has to be controled by the conditions defined for the system. This assumption seems acceptable in the light of the finding of Berezin et al. [14] that particles are created in pairs of positive energy by the black hole, where one is emitted to infinity and the other falls on the black hole causing a change of the inner structure. Then, we immediately deduce that the new event horizon will

have a radius of $\frac{4}{3}R_s$, and so the growth of the event horizon will go on. This means that

$$R_n = \left(\frac{4}{3}\right)^n R_s, \quad (20)$$

where $n = 1, 2, 3, \dots$

The argument we place for considering discrete eigen values for the radius of the event horizon is simple and goes as follows: The first Casimir system which is composed of the event horizon and the barrier surface at $R_1 = \frac{4}{3}R_s$ will create an amount of positive energy (the Casimir energy) once formed, then if this energy is assumed to be added to the total energy of the black hole, the event horizon will be extended to a new position and the second surface of the new Casimir system (the new position of the barrier) will be at $R_2 = \frac{4}{3}R_1$ and so it goes.

According to the above mechanism, the surface area of the event horizon of the black holes will grow as

$$A_n = \left(\frac{4}{3}\right)^{2n} A_s. \quad (21)$$

This means that the area of the event horizon is quantized. The quantization law here is much different from the law deduced by Bekenstein and Mukhanov [17] according to which the spectrum of the surface area of the event horizon was uniformly spaced. However, using the loop representation of quantum gravity, Barreira et al. [8] have shown that the Bekenstein-Mukhanov area quantization spectrum is unrecoverable, consequently they deduce that the Bekenstein-Mukhanov result is likely to be an artefact of the ansatz used rather than a robust result.

From (20) it is clear that the expansion of the black hole will be logarithmic (i.e inflationary). The number of the inflationary foldings is given by

$$n = 8 \log \left(\frac{R_n}{R_0} \right) \quad (22)$$

VI. A Constant-Time Hypersurface (Cauchy Surface) Structure

Using the result in (20) above, we can construct a hypothetical concentric Cauchy surface structure centered at the black hole singularity. This structure is characterized by the eigenvalues of (21) and a set of time-like Killing vectors normal to the surfaces. The transition from one surface to another is associated with translation in time, and consequently generation of energy. Therefore each surface will represent an energy level characterizing a black hole of the corresponding mass. The relative temporal separation between these surfaces is given by the basic relation

$$\frac{t(x_1)}{t(x_2)} = \left[\frac{g_{00}(x_1)}{g_{00}(x_2)} \right]^{-1/2}. \quad (23)$$

It is clear that the temporal separations between surfaces situated near to R_s are larger than those far away. If x_2 is a point at the asymptotically-flat region where $t(x_2) \equiv t_\infty$, then we can write

$$\begin{aligned} t(x_n) &= t_\infty [g_{00}(x_n)]^{-1/2} \\ &= t_\infty \left[1 - \left(\frac{3}{4} \right)^n \right]^{-1/2}. \end{aligned} \quad (24)$$

This structure represents an infinite set of hypothetical Cauchy concentric spherical surfaces surrounding the black hole prior to the natural development of the black hole's event horizon. However, if the positive energy created by the black hole within the specified region is to be added to the black hole total energy, then the mass development will take the following form

$$M_n < \left(\frac{4}{3} \right)^n M_0, \quad (25)$$

where M_0 is the initial mass of the black hole.

Energy levels of the above structure have the following separations

$$E_{n+1} - E_n < \frac{1}{3} \left(\frac{4}{3} \right)^n E_0, \quad (26)$$

where E_0 is the total initial energy of the black hole.

From (20) and (25) we find that the energy (mass) density of the system will develop according to the inequality

$$\rho_n > \left(\frac{3}{4}\right)^{2n} \rho_0, \quad (27)$$

where ρ_0 is the initial density. This means that

$$\log\left(\frac{\rho_n}{\rho_0}\right) > -0.2498n. \quad (28)$$

This equation determines the development of the density of the non-rotating black hole under the condition that the created energy is added to its initial energy.

VII. Cosmological Applications

If the universe was born as a singularity, then it is unknown how it has crossed its own event horizon. Although quantum effects may dissipate the creation singularity, the presently available calculations, which incorporate quantum fields into the classical curvature, do not indicate the possibility of a universe born with crossed event horizon. The available calculations [21,22,23] indicates that the universe was born as a finite-sized patch with dimensions less than the Schwartzchild radius. This implies that the universe may have been born as a black hole and is still is. This idea is not new, and there are a number of investigations that support it; for example it was already shown long ago by Oppenheimer and Snyder [24] that the inside of the Schwartzchild solution could be a Friedmann universe. Moreover it was shown by Pathria [25] that our present universe may be described as an internal Schwartzchild solution if it has the critical energy density. More recent investigations [26] based on the assumption of the existence of a limiting curvature have shown that the inside of a Schwartzchild black hole can be attached to a de Sitter universe at some space-like junction which is taken to represent a short transition layer. Other scenarios in which the universe emerges from the interior of a black hole are also proposed [27-33].

We will now utilize the results of the previous section to construct a model for the whole universe. The model adopts the results of previous calculations [21,23] of the back-reaction of the finite temperature corrections to the vacuum energy density of the photon field in an Einstein universe. Although the Einstein universe is static, the conformal relation with the closed Robertson-Walker universe [34] allow us to consider the results as being of practical interest. In fact, the discrete spectrum provided here by the inflating black hole model can be considered as representing instantaneous successions of different states of the Einstein static universe. The results of the back-reaction calculations have shown that the thermal development of the universe covers two different regimes; the Casimir regime, which extends over a very small range of the radius but huge rise of temperature from zero to a maximum of $1.44 \times 10^{32} \text{K}$ at a radius of $5.5 \times 10^{-34} \text{cm}$. At this maximum temperature a phase transition takes place, and the system cross-over to the Planck regime, where photons get emitted and absorbed freely exhibiting pure black-body spectrum. At this point one can identify the primordial universe with an initial energy density ρ_i given by

$$\begin{aligned}\rho_i &= \alpha T^4 \\ &= 3.2528 \times 10^{114} \text{erg/cm}^3.\end{aligned}\tag{29}$$

However, the same calculations [23] showed that the Einstein universe exhibit the presently measured microwave background temperature of 2.73K at a radius of $1.83 \times 10^{30} \text{cm}$. Consistency require us to adopt this value for the radius of the present universe. Therefore, from (22) we can write

$$n = 508.176.\tag{30}$$

Accordingly, we deduce from (28) and (30) that

$$\log \frac{\rho_n}{\rho_i} \approx -126.9425.\tag{31}$$

Using the value in (29) for ρ_i we get

$$\rho_n = 3.7133 \times 10^{-13} \text{erg/cm}^3.\tag{32}$$

This result is very close to the radiation density in the present universe, calculated in reference to the cosmic microwave background.

One may argue that the estimated radius of the present universe (Hubble length) is $1.38 \times 10^{28} cm$ and not $1.83 \times 10^{30} cm$. In such a case we find that

$$n = 508.176. \quad (33)$$

Therefore, from (28) we have

$$\log \frac{\rho_n^*}{\rho_i} \approx -122.773, \quad (34)$$

which means that

$$\rho_{now}^* \approx 6.0956 \times 10^{-30} gm.cm^{-3}. \quad (35)$$

a figure which is very close to the critical matter density which defines a flat universe.

VIII. Discussion and Conclusions

The present investigation tackled many aspects of the subject of particle creation by localized gravitational fields. The aim was to utilize the basic concepts and principles of the general theory of relativity and quantum mechanics and present a simple approach to deduce particle creation by Schwartzchild black holes. The analysis led us to determine an upper limit for the region, in the vicinity of the black hole, where real quanta can be produced. On one hand this result comes in support to the school of thought which suggests that the quanta of the Hawking effect are created in the vicinity of the black hole [35], and on the other hand this result confine the creation region into a spherical shell of thickness $\frac{1}{3}R_s$ outside the event horizon. We emphasize here that the regional upper limit is consistent with the results of other authors [9,10,12].

The simple approach followed in this paper led us to a new quantization law for the area of the event horizon, and consequently into a new area spectrum. The main features of the new spectrum is that it is logarithmic and macroscopic, in contrast to the Bekenstein-Mukhanov spectrum [15], which

was linear and microscopic (i.e, Planck dimensional). In fact the Bekenstein-Mukhanov spectrum cannot be verified observationally because in practice the spectrum will look continuous for macroscopic black holes.

A logarithmic inflation arises naturally in our model as a result of the Casimir system assumption. Normally such a model will benefit from all the privileges of inflationary models over the standard big bang model, less their conceptual problems. This is indeed the case since it was recently shown by Easson and Brandenberger [27] that a universe born from the interior of a black hole will not possess many of the problems of the standard big bang model. In particular the horizon problem, the flatness problem and the problem of formation of structures are solved naturally. This may well be the case for a universe formed of the interior of an inflating black hole. Perhaps this is the most important result that needs to be analyzed further to see if one can draw some observational consequences.

Our assumptions in this paper are strongly supported by the results we obtained for matter and radiation densities in the present universe. One can see clearly that starting with a Planck-dimensional black hole universe, the mechanism of the quantum development of such a hypothetical universe leads to a universe having the present critical matter density, a point which is supported by the recent observational investigations of the Boomerang project [36]. Further analysis and development of this approach by investigating a Kerr or Reissner-Nördstrom black holes will be interesting, where one may expect the emergence of a different scheme for area quantization.

IX. References

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